J. Fluid Mech. (1983), *vol.* 126, *pp.* 369–378
Printed in Great Britain

The settling of a porous body through a density interface

By S. A. THORNHAM AND S. D. R. WILSON
Department of Mathematics, University of Manchester

(Received 24 April 1982)

In a large-scale process at I.C.I. Mond Division a flocculated suspension of fine solid particles is allowed to settle through an aqueous solution towards the horizontal interface with an underlying denser solution. When the solids have settled into the lower layer they are pumped away and dumped while the clarified upper layer can be safely disposed of or recycled. The passage of the flocs through the interface is the rate-determining step in the process, and the analysis of this part of it is the subject of the paper. It is hoped that the analysis may have a much wider application in that the settling process could be used as the basis of a direct laboratory method of measuring properties of flocs. At present both the porosity and permeability of flocs are almost impossible to measure directly, but, using the present analysis, the settling of a single floc through two successive density interfaces would provide (in principle) values for both.

1. Introduction

A common problem in mineral processing, and chemical engineering in general, is the disposal of very large quantities of dirty water; that is, water containing a dilute suspension of very fine solid particles. The normal practice is to introduce a small quantity of a long-chain polymer into the water in order to flocculate the solid contaminant; the polymer binds the particles into loose, irregular, open clumps, or flocs, which settle much faster than the individual particles. The flocculated slurry is usually pumped into a large tank and allowed to settle, a process known as gravity thickening which has been much studied (Pearse 1977).

The clarified water can then be safely disposed of, while the thickened sludge has to be filtered, washed and finally buried somewhere. In a large-scale process at I.C.I. Mond Division the sludge was conveyed away by means of a second stream of waste liquor. However, this second stream of water is cooler and contains more dissolved salts than the original liquor from which the sludge came, and therefore is denser, and it was realized that the troublesome and expensive filters could be eliminated by exploiting the density difference. The original lighter liquor, containing flocculated solids, is arranged to lie over a layer of the more-dense liquor in a large tank, and the flocs simply settle from one to the other, the interface itself acting as a filter.

It is not difficult to see that the rate-determining step in this process is the passage of the flocs through the interface between the two liquids. In actual operation the flocs form a fairly homogeneous layer just above the interface, and arrangements are made to keep this fairly well broken up and dispersed. As a first step towards understanding and optimizing the process it is desirable to analyse the process by which a single floc (whose linear dimensions are of the order of millimetres) passes through a horizontal density interface, and this is the purpose of the present paper.

In §2 the model will be described and reduced to a well-defined boundary-value

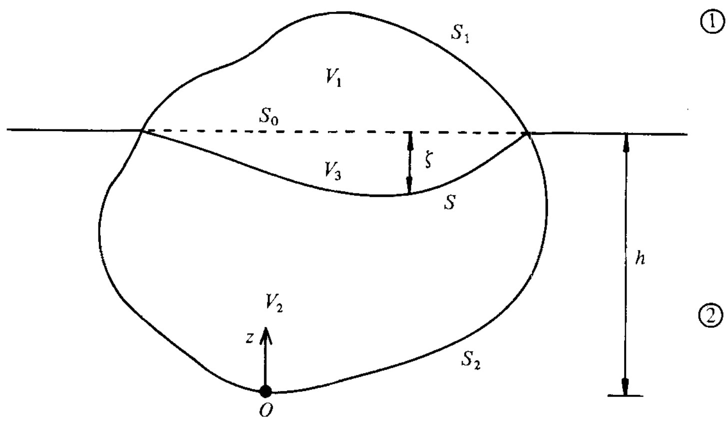

FIGURE 1. A porous solid in the interface between fluids (1) and (2). The interface is undisturbed outside the body but is deformed downwards inside to provide the buoyancy force.

problem, which is in general a nonlinear free-boundary problem. We then develop two approaches to this. First, under certain rather artificial constraints the equations can be made one-dimensional (i.e. reduced to ordinary differential equations) so that the nonlinearity can be handled. This simple treatment will help to explain the basic physics and also reveal the correct non-dimensional variables. It turns out that a parameter emerges which is likely to be small in practice, and this can be used to obtain a linearized version of the problem. The second approach consists of the solution of this linear problem without the constraints mentioned above, and in which the equations are therefore partial differential equations.

2. Description of the model

Suppose we have a region of liquid consisting of two layers of different densities separated by a horizontal interface, the less-dense liquid 1 (of density ρ_1) overlying the more-dense liquid 2 (of density ρ_2). A porous body, assumed rigid, sinks through the upper liquid towards the interface. The pores are filled with liquid 1, and so this sinking may be fairly rapid, since the weight of the body is balanced only by hydrodynamic drag at the terminal speed.

When the body reaches the interface it cannot continue at the same rate because buoyancy forces will act on it and slow it down. The configuration is sketched in figure 1. Here the body is sinking slowly while liquid 2 penetrates the surface of the body, percolates through the pores and displaces liquid 1. (It is assumed that liquid 1 is completely displaced, that is, there are no closed pores. This is reasonable because the flocs are very open in practice.) We assume that outside the body the density interface is undisturbed; inside it is displaced downwards of course, and this is what provides the buoyancy force. Here we suppose that the hydrodynamic drag is negligible and that the weight of the body is balanced by the buoyancy force. (The term 'balanced' here means that the drag and the mass-acceleration are negligible by comparison, not of course that the body is literally motionless.) We cannot, of course, pass to this balance from that described in the previous paragraph without a transition regime in which drag and buoyancy are both important, but this will not be studied.

The two liquids are both dilute aqueous solutions, as noted above, so there will be no interfacial tension in the usual sense, and no interfacial effects are included in the model. However, the liquids will be regarded as immiscible in the sense that any diffusion of solute or heat will be supposed to occur on a timescale longer than any timescale of interest. We may expect the interface to remain smooth and not to exhibit the well-known 'fingering' instability because the denser more-viscous liquid is advancing.

The floc will be regarded as rigid even though flocs are extremely fragile by everyday standards. Indeed this fragility is what makes them difficult to study in the laboratory and to handle in industrial practice – a layer of flocs a few centimetres thick would, if brought to rest, be significantly compressed by its own weight, with a consequent marked decrease in permeability and the formation of closed pores. However there was no evidence in laboratory experiments that flocs were distorted by their passage through the density interface, and this process may in fact provide the basis of a laboratory technique of holding flocs more or less stationary with the minimum of applied force, so that their properties can be measured. (The passage through the interface occupied a few minutes in the particular case of interest.)

We begin by considering the overall equilibrium of the body. In figure 1, V_1 and V_2 are the volumes occupied by fluids 1 and 2 respectively, $V = V_1 + V_2$ is the total volume, and V_3 is that portion of V_1 below the level of the undisturbed interface. We take Cartesian coordinates $Oxyz$ fixed in the body, with z measured vertically upwards. The interface S between the two fluids can be described by the equation

$$z = h(t) - \zeta(x, y, t), \tag{1}$$

so that $h(t)$ is essentially the depth to which the body has penetrated into the lower fluid, and $\zeta(x, y, t)$ describes the interface. Then we have

$$V_3 = \iint \zeta \, dx \, dy, \tag{2}$$

where the integration is carried out over S_0, the horizontal section through the body at the level of the exterior interface. The porosity η of the body (the volume fraction occupied by fluid) is assumed uniform. Then equating the total weight of the body to the upthrust of its surroundings (Archimedes principle), we have

$$\rho_s(1-\eta)\, V + \rho_2 V_2 \eta + \rho_1 V_1 \eta = \rho_1(V_1 - V_3) + \rho_2(V_2 + V_3), \tag{3}$$

and this can be rearranged to give

$$(1-\eta)\{\rho_s V - \rho_2(V_2 + V_3) - \rho_1(V_1 - V_3)\} = (\rho_2 - \rho_1)\, \eta V_3. \tag{4}$$

Here ρ_s is the 'true' density of the solid; that is, the density the body would have if there were no pores in it at all.

Next we suppose that in this motion through the pores the fluid obeys Darcy's law, so that, using subscripts ($i = 1, 2$) to denote the two fluids, we have

$$\frac{\mu}{\lambda}\mathbf{v}_i = -\text{grad}\, p_i - \rho_i \mathbf{g}, \tag{5}$$

$$\text{div}\, \mathbf{v}_i = 0. \tag{6}$$

Here we are assuming for simplicity that the viscosity μ is the same for the two fluids, and the permeability λ is then a constant of the body and is assumed uniform. The velocities $\mathbf{v}_i$ are mean filter velocities (i.e. they measure flux per unit area) and are not the same, of course, as the mean microscopic velocities $\mathbf{u}_i$, say. The two are related by

$$\mathbf{v}_i = \eta \mathbf{u}_i. \tag{7}$$

The importance of $\mathbf{v}_i$ is that it relates to the sinking speed dh/dt, whereas $\mathbf{u}_i$ measures the speed with which the interface moves through the body.

It is convenient to introduce the reduced pressures P_i defined by

$$P_i = p_i + \rho_i g(z-h) \quad (i = 1, 2), \tag{8}$$

so that (5) and (6) reduce to

$$\nabla^2 P_i = 0 \quad (i = 1, 2). \tag{9}$$

On the exterior boundary of the body we have the conditions

$$P_i = 0 \quad \text{on } S_i \quad (i = 1, 2), \tag{10}$$

and we now turn to the conditions at the interior interface S. Here the true pressure p and the normal velocity are continuous, so that

$$\left.\begin{aligned} P_2 - P_1 &= -\Delta\rho g \zeta \\ \frac{\partial P_2}{\partial n} - \frac{\partial P_1}{\partial n} &= 0 \end{aligned}\right\} \quad \text{on} \quad z = h-\zeta, \tag{11}$$

where n is the normal to S and $\Delta\rho = \rho_2 - \rho_1$. Finally we have the kinematic condition

$$\left(\frac{\partial}{\partial t} + \frac{1}{\eta}\mathbf{v}.\ \mathrm{grad}\right)(z-h+\zeta) = 0 \quad \text{on} \quad z = h-\zeta, \tag{12}$$

and here $\mathbf{v}$ can be either $\mathbf{v}_1$ or $\mathbf{v}_2$.

We now consider how the solution could be obtained in principle from the above system of equations and boundary conditions – more precisely, how, given the current values of h and ζ, the solution can be advanced by a small time increment. If h and ζ are known we can solve (9), (10) and (11) to determine P_1 and P_2 and hence $\mathbf{v}_1$ and $\mathbf{v}_2$. Then (12) yields an equation connecting $\dot{h}$ and $\dot{\zeta}$. Carrying out the integration indicated in (2) and differentiating (4) with respect to t, we can eliminate $\dot{\zeta}$ and obtain an equation connecting h and $\dot{h}$ from which the increment in h can be found. Finally the change in ζ can be computed from (12).

Evidently a solution obtained in this way cannot be made to satisfy arbitrary initial conditions, a defect which is inevitable under the quasistatic approximation. So what we are assuming is that the solution of the full problem rapidly becomes independent of the initial conditions, or at any rate of the details thereof. It will turn out that in the actual solutions we obtain below this difficulty can be sidestepped. We measure the time from the first moment when the assumed balance of forces can hold and assume that at this time the body has penetrated the interface by a sufficient distance for the buoyancy forces to support its weight but that the lower fluid has not had time to penetrate appreciably into the pores. This is expressed in the initial condition

$$V_2 = 0 \quad (t = 0). \tag{13}$$

This implies that $V_1 = V$ of course, and then (3) can be used to determine the initial value of V_3. This is

$$V_3(0) = \frac{(1-\eta)(\rho_s - \rho_1)V}{\Delta\rho}. \tag{14}$$

Now $V_3(0)$ must be less than V if the proposed model is to make any sense, and this restriction can be rearranged as

$$\eta > \frac{\rho_s - \rho_2}{\rho_s - \rho_1}. \tag{15}$$

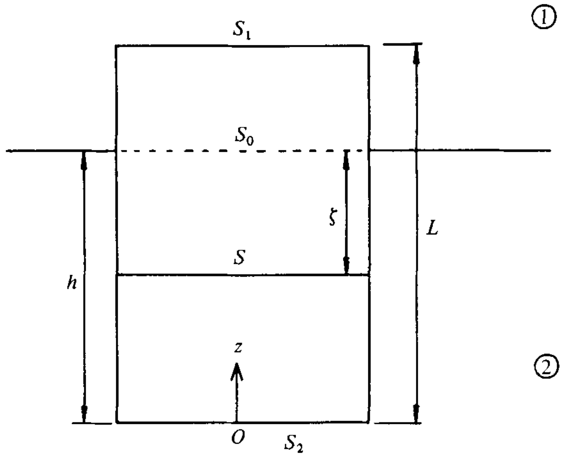

FIGURE 2. One-dimensional model. The vertical faces of the body are impermeable and the interface S is a horizontal plane.

This result is interesting because it indicates that if η can be made small enough to violate it the flocs will pass straight through the interface without pausing. On the other hand, it will turn out that in this case the flocs carry through with them the maximum volume of the upper fluid and it may be desirable to reduce this contamination.

3. One-dimensional model

Here we suppose that the body has the form of a cylinder with vertical generators and with impermeable sides (figure 2). The fluid can leave or enter the body only by the horizontal top and bottom surfaces, and clearly all quantities can depend only on z and t.

It is a simple matter to solve for P_1, P_2 and the velocity v (which is the common value of v_1 and v_2), with the result

$$v = \frac{\lambda \Delta\rho g \zeta}{\mu L}. \tag{16}$$

The kinematic condition (12) becomes

$$-\dot{h} + \dot{\zeta} + \frac{1}{\eta} v = 0, \tag{17}$$

and differentiating (4) with respect to t gives

$$-(1-\eta)\dot{h} = \eta\dot{\zeta}. \tag{18}$$

The initial condition is obtained from (14) and takes the form

$$\zeta = \zeta_0 = \frac{(1-\eta)(\rho_s - \rho_1) L}{\Delta\rho} \quad (t = 0), \tag{19}$$

and so we find

$$\zeta = \zeta_0 \exp\left\{\frac{-\lambda g \Delta\rho(1-\eta)t}{\mu L \eta}\right\}, \tag{20}$$

and also

$$h = \frac{\zeta_0}{1-\eta}\left[1-\eta\exp\left\{\frac{-\lambda g\Delta\rho(1-\eta)\,t}{\mu L\eta}\right\}\right]. \tag{21}$$

As t increases, h approaches a maximum value $\zeta_0/(1-\eta)$, and this must exceed L if the body is eventually to sink. Using (19) we can see that this will hold if $\rho_s > \rho_2$, which is merely the condition that the body is negatively buoyant in the lower fluid when its pores are filled with that fluid. Given this, the body will leave the vicinity of the interface soon after the moment when $h = L$, and the time T that will have elapsed satisfies

$$\exp\left\{\frac{-\lambda g\Delta\rho(1-\eta)\,T}{\mu L\eta}\right\} = \frac{1}{\eta}\frac{\rho_s-\rho_2}{\rho_s-\rho_1}, \tag{22}$$

and the right-hand side is less than unity by (15).

The volume of the upper fluid carried through is proportional to $\eta\zeta(T)$, and this is given by

$$\eta\zeta(T) = L(1-\eta)\frac{\rho_s-\rho_2}{\Delta\rho}. \tag{23}$$

Now clearly if a short transit time is required η should be made small (from (22)), but this maximizes the contamination of the lower fluid by the upper (from (23)).

This simple analysis also suggests the important idea that, if η is sufficiently close to 1, ζ will be small compared with L (from (19)) and this will permit the interface conditions of §2 to be linearized, and we now turn to this problem.

4. Linearized model

We now introduce suitable dimensionless variables. We suppose that L denotes a typical linear dimension of the body, and use this as the scale for x, y, z and h. Then V, V_1 and V_2 are of order L^3. The correct scale for ζ, say l, can be determined from (4) and turns out to be $l = \epsilon L$, where

$$\epsilon = \frac{(1-\eta)(\rho_s-\rho_1)}{\rho_2-\rho_1}. \tag{24}$$

We are supposing that $\epsilon \ll 1$, which requires $\eta \approx 1$ as noted. The factor involving the densities would not of course make any difference, strictly speaking, to an asymptotic solution valid as $\eta \to 1$, but is in practice likely to be quite a large number (10 or so) and places a limitation on the usefulness of the results.

Using an asterisk to denote dimensionless versions of variables, we can rewrite (4), correct to leading order in ϵ, as

$$V^* - \kappa V_2^* = V_3^*, \tag{25}$$

where κ is the density factor in (24),

$$\kappa = \frac{\Delta\rho}{\rho_s-\rho_1}, \tag{26}$$

and will normally take small values.

Next, the scale for P_1 and P_2 is determined from (11) to be $\Delta\rho g l$, and the dimensionless version of (11) is

$$P_2^* - P_1^* = -\zeta^*, \quad \frac{\partial P_2^*}{\partial n^*} - \frac{\partial P_1^*}{\partial n^*} = 0 \quad \text{on} \quad z^* = h^* - \epsilon\zeta^*. \tag{27}$$

The timescale is determined from the kinematic condition (12) to be $\mu L/\lambda g \Delta\rho\epsilon$, and the dimensionless version of this equation is, to leading order in ϵ,

$$w^* = \frac{dh^*}{dt^*} \quad \text{on} \quad z^* = h^* - \epsilon\zeta^*. \tag{28}$$

It is a simple matter to approximate (27) and (28) further by neglecting the displayed terms in ϵ, thereby transferring the conditions to the plane surface $z^* = h^*$. (No terms in ϵ will be required here.) However, there is a crucial further simplification implicit in (28), namely that w^* is a constant on $z^* = h^*$. This constant (or rather, function of t^* only) is the common value of $\partial P_1^*/\partial n^*$ and $\partial P_2^*/\partial n^*$ on the interface, and, since P_1^* and P_2^* satisfy linear boundary-value problems, it can be carried through the analysis as a factor. Then ζ^* can be found from the first equation of (27) and then V_3^* from the expression

$$V_3^* = \iint \zeta^* \, dx^* \, dy^*, \tag{29}$$

the integration being carried out over the plane $z^* = h^*$. Finally we can use (25) to connect $w^* = \dot{h}^*$ to h^*.

In §5 we present the solutions for some particular cases.

5. Sample calculations

5.1. *Rectangle*

Here we give probably the simplest extension of the one-dimensional model, allowing percolation in at the sides, in order to illustrate the method. The rectangle is of unit width and height α and the flow is two-dimensional. In this case (25) can be written

$$\alpha - \kappa h^* = \int_0^1 \zeta^* \, dx^*. \tag{30}$$

The functions P_1^* and P_2^* can be obtained as Fourier series:

$$P_1^* = \frac{4}{\pi^2} h^* \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \frac{\sinh (2n-1)(\alpha - z^*)\pi}{\cosh (2n-1)(\alpha - h^*)\pi} \sin (2n-1)\pi x^*, \tag{31a}$$

$$P_2^* = -\frac{4}{\pi^2} h^* \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \frac{\sinh (2n-1) z^* \pi}{\cosh (2n-1) h^* \pi} \sin (2n-1)\pi x^*, \tag{31b}$$

Now from (27) we find ζ^*, and the integral in (30) is easily evaluated to give

$$\alpha - \kappa h^* = \frac{8}{\pi^3} h^* \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \{\tanh (2n-1)(\alpha - h^*)\pi + \tanh (2n-1) h^* \pi\}, \tag{32}$$

which must be solved numerically. The initial condition to leading order in ϵ is

$$h^*(0) = 0, \tag{33}$$

and the quantity of interest, the time taken for the body to sink, T^* say, is given by the condition $h^*(T^*) = \alpha$. Thus

$$T^* = \frac{8}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \int_0^{\alpha} \frac{\tanh (2n-1)(\alpha - \xi)\pi + \tanh (2n-1)\xi\pi}{\alpha - \kappa\xi} d\xi. \tag{34}$$

κ	1-dimensional $\alpha = 1$	Rectangle, $\alpha = 1$	Rectangle, $\alpha = 2$	Rectangle, $\alpha = 3$	Circle	Sphere
0·05	1·026	0·4385	0·4974	0·5172	0·7996	0·5344
0·10	1·054	0·4503	0·5108	0·5311	0·8211	0·5488
0·15	1·083	0·4630	0·5252	0·5461	0·8442	0·5642
0·20	1·116	0·4766	0·5407	0·5623	0·8690	0·5809

TABLE 1

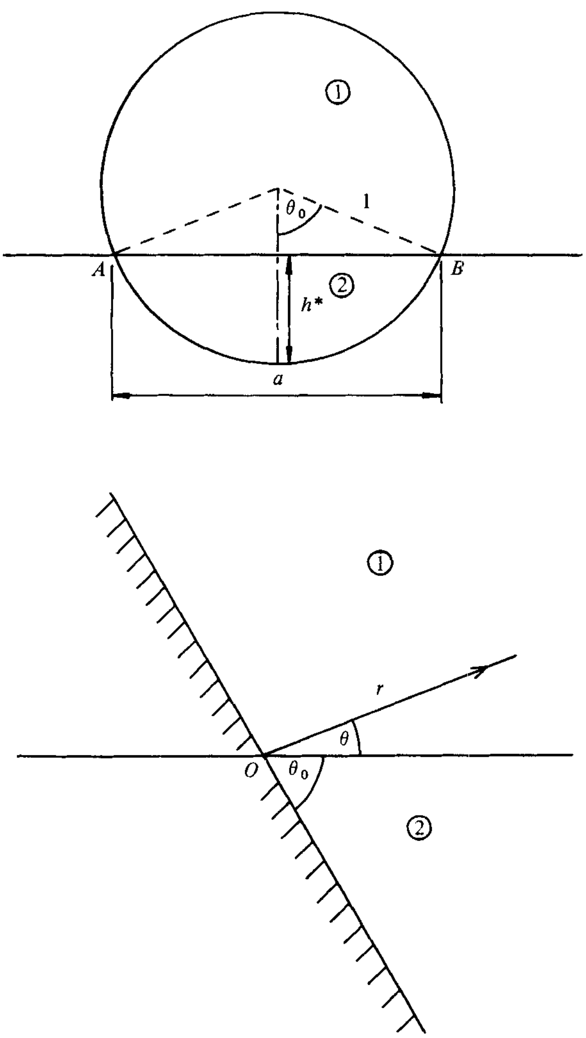

FIGURE 3. Circular cylinder with horizontal generators. The interior is mapped conformally onto the half-plane in the lower sketch, the interface AB mapping onto the line $\theta = 0$. The upper sketch also gives the notation for a spherical body, in which case it represents a vertical section through the centre.

Sample results are given in table 1. In this table are also given the results for the one-dimensional model in the present dimensionless variables. That problem has of course the simple exact solution

$$T^* = -\frac{\alpha}{\kappa}\ln(1-\kappa). \tag{35}$$

5.2. *Horizontal circular cylinder*

This problem is sketched in figure 3, which also shows the half-plane on to which the interior of the circle (of unit radius) will be mapped conformally in order to solve the potential problem. It is convenient to introduce the angle θ_0 and the width a, of the interface, which are connected to h^* by the equations

$$\cos\theta_0 = 1-h^*, \tag{36a}$$

$$\sin\theta_0 = \tfrac{1}{2}a = (2h^*-h^{*2})^{\frac{1}{2}}. \tag{36b}$$

Then (25) takes the form

$$\pi-\kappa(\theta_0-\sin\theta_0\cos\theta_0) = \int_{-\frac{1}{2}a}^{\frac{1}{2}a}\zeta^*\,dx^*. \tag{37}$$

To find P_1^* and P_2^* we first map the interior of the circle on to a half-plane, as noted above; the point A in the physical plane corresponds to the origin in the transform plane, and the interface to the line $\theta = 0$. On this line we have the boundary condition

$$\frac{\partial P_i^*}{\partial\theta} = -ah^*\frac{r}{(1+r)^2} \quad (i = 1, 2), \tag{38}$$

and the other conditions are

$$P_1^* = 0 \quad (\theta = \pi-\theta_0), \tag{39a}$$

$$P_2^* = 0 \quad (\theta = -\theta_0). \tag{39b}$$

This problem is conveniently solved by means of the Mellin transform, and we shall not give any further details here. It is not necessary to invert the separate transforms to find P_1^* and P_2^* because all we require is the integral of the difference, which is the right-hand side of (37). This can be written

$$\pi-\kappa(\theta_0-\sin\theta_0\cos\theta_0) = a\int_0^\infty\frac{\zeta^*\,dr}{(r+1)^2}$$

$$= a^2\pi h^*\int_0^\infty\frac{y\{\tanh y(\pi-\theta_0)+\tanh y\theta_0\}\,dy}{\sinh^2\pi y}. \tag{40}$$

Finally, with the initial condition $h^*(0) = 0$ and the sinking time T^* defined by $h^*(T^*) = 2$ we find

$$T^* = 4\pi\int_0^2\frac{\sin^2\theta_0\,dh^*}{\pi-\kappa(\theta_0-\sin\theta_0\cos\theta_0)}\int_0^\infty\frac{y\{\tanh y(\pi-\theta_0)+\tanh\theta_0\}}{\sinh^2\pi y}dy. \tag{41}$$

The repeated integral can be easily evaluated numerically and the result is in table 1.

5.3. *Sphere*

For this we can refer to figure 3, which is now interpreted as a vertical section through the centre. Equation (25) takes the form

$$\tfrac{2}{3}-\tfrac{1}{2}\kappa(h^{*2}-\tfrac{1}{3}h^{*3}) = \int_0^{\frac{1}{2}a}\zeta^* r^*\,dr^*, \tag{42}$$

in which r^* is the distance from the vertical axis of symmetry.

The problem of determining P_1^*, P_2^* and the integral in (42) can be solved using toroidal coordinates and the Mehler–Fock transform (Sneddon 1972). As in the previous example the analysis can be pressed to a reasonably manageable conclusion because we do not need to invert the separate transforms for P_1^* and P_2^* but only to evaluate the integral in (42). When this is done we find that (42) becomes

$$\tfrac{2}{3}-\tfrac{1}{2}\kappa(h^{*2}-\tfrac{1}{3}h^{*3}) = \sin^3\theta_0\, h^* \int_0^\infty \frac{\tau^2\{\tanh\tau(\pi-\theta_0)+\tanh\tau\theta_0\}\,d\tau}{\sinh\pi\tau\,\cosh\pi\tau}. \tag{43}$$

Since the initial and final conditions are as in the previous example, we find that the sinking time T^* is given by

$$T^* = 12\int_0^2 \frac{(2h^*-h^{*2})^{\frac{3}{2}}\,dh^*}{1-\frac{1}{4}\kappa(3h^{*2}-h^{*3})}\int_0^\infty \frac{\tau^2\{\tanh\tau(\pi-\theta_0)+\tanh\tau\theta_0\}}{\sinh\pi\tau\,\cosh\pi\tau}\,d\tau. \tag{44}$$

Again the integral is easily evaluated numerically, and the results are in table 1.

6. Concluding remarks

The analysis here represents a first step towards understanding the settling of a flocculated suspension through a density interface. From the practical point of view the order-unity numbers generated by solving the model equations are of less interest than the non-dimensionalization itself, and in the timescale for settling in particular. This has been shown to be

$$\frac{\mu L}{\lambda g\Delta\rho\epsilon} = \frac{\mu L}{\lambda g(1-h)\,(\rho_s-\rho_l)}. \tag{45}$$

Some experiments have been performed in the laboratories at I.C.I. Mond Division, but a meaningful comparison with the model is not possible because of the difficulty (or near-impossibility) of measuring the permeability λ. It may be possible to measure the porosity η – a method has been proposed by Akers (1980) – but laboratory methods of measuring λ are suitable only for rigid materials such as rock or granular packed beds. It may in fact turn out that the passage of a floc through a density interface will provide the only reasonable way of measuring λ, as it were reversing the roles.

Clearly the value of the settling time through an interface provides one equation connecting λ and η (provided of course that the size of the floc can be independently measured) and so the passage of a single floc through two successive density interfaces would provide (in principle) enough information to determine λ and η.

We have also shown that if the porosity falls below a certain critical value (given by (15)) the flocs will sink through the interface in a much shorter time but carrying through more of the upper liquid. This may cause undesirable contamination, and furthermore it appears that to achieve such low porosities would require the addition of large amounts of the flocculating agent, which adds considerably to the costs.

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